Homework 4

February 14th
Due February 21st

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name, class number and assignment number should be clearly visible (like on this document for example). Multipage assignment must be stapled.

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not. I will be available at my office hours[1] to help you through your reading, or if you need clarifications on an exercise statement.

Reading

The class textbook is *Notes on Elementary Probability,* by Liviu I. Nicolaescu. You will find it on the course website.

Section 1.3. With the exception of Proposition 1.43, Remark 1.44, Example 1.51, Example 1.53 (b) and Example 1.57, this section is required reading for this homework. Of course you can skip the parts that were advised in previous homeworks if you read through it already.

Exercises from the book

Included in this homework are exercises 1.34, 1.36, 1.39, 1.40, 1.42, 1.46, 1.48 and 1.52.

Exercise 1

This is a variation on exercise 1.74 from the book.

An isolated research station sends its tutor government agency two signals each morning. Each of them can be "OK" or "Error", referring to the stability of containment chambers 1 and 2. However, the signal is not perfectly reliable, and there is a 1% chance that a given signal is received as its opposite ("Error" instead of "OK" and vice versa), independently of the other.

1. If the station sends "OK, OK", what is the probability that "OK, OK" is received?

[1] Monday and Thursday, from 10:00 to 11:30, or by appointment.

2. There is a minor problem about one day out of five for a given chamber, randomly and independently of the other chamber.

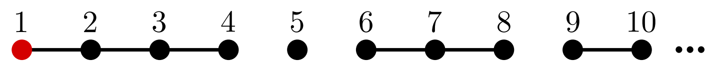
   (a) What is the probability that the station sends "OK, Error" on a given day?

   (b) The agency receives "OK" for containment chamber 1. What is the probability that there is a problem in chamber 1?

   (c) The agency receives "OK, OK". What is the probability that, actually, all is not OK?

Exercise 2

On a half-line, see below, we plot regularly spaced points; think of it as $\mathbb{N}^*$ if you like. Between two consecutive points, we draw an edge with probability $0 < p < 1$, independently from each other, and we can go from point to point only if there is an edge between these two.



1. Call N the index of the farthest point we can reach, starting from the leftmost point. During class, we gave a name to the distribution of N; what is it?

2. Show that the probability that we can go from 1 to infinity is zero.

   Hint: the event 'We can reach infinity' is included in the event 'We can reach 10,000'.

3. Consider the same model, but on the whole axis $\mathbb{Z}$ (infinite in both directions). What is the probability that we can go from zero to infinity, either to the left or to the right?