Homework 10

April 23rd
Due April 30th

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name, class number and assignment number should be clearly visible (like on this document for example).

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not.

Exercises from the book

Included in this homework are exercises 2.32, 2.34,[1] 2.36.

Exercise 1

This exercise is a detailed version of exercise 2.33 in the book.

You want an object as a silent auction: you secretly bid a certain amount, as do the other potential buyers, and the highest bid takes the prize and while others just go home with no less money and no antique cabinet. You estimate that the maximum bid from others is uniform in $[70, 130]$, so this is the number you are trying to beat.

1. If your bid is x, what is your probability of winning the auction?

2. If you win the object, you can sell it for 100$. Assuming again that you bid x is fixed, what is the expected gain you will make?

3. What is the bid you should make to have the highest expected gain?

Exercise 2

The following graph shows the probability density function of a continuous random variable X.

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[1] At random here means uniformly along the stick.
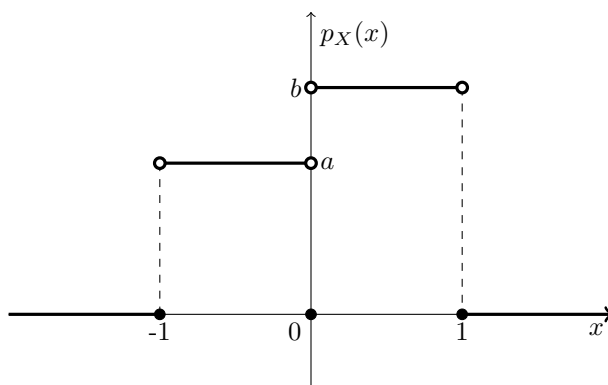
1. What are the possible values for a and b?

2. For which values of a and b is the variance minimal?

Exercise 3

This is actually Exercises 2.37 and 2.45 of the book, with additional hints. For both exercises, follow the following steps.

Define X and Y as in the book.

1. What is the cumulative distribution function of X?

2. What is the cumulative distribution function of Y? You can try to rewrite the event $\{Y \leq y\}$ in terms of X.

3. What is the probability density function of Y?

As the exercise notes, in the case of Exercise 2.37, the density of Y should be that of an exponential variable of parameter 1.