

Homework 4

March 5th
Due March 12th

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name and assignment number should be clearly visible (like on this document for example). It is due on Gradescope.

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not. I will be available at my office hours¹ if you need help.

Exercises from the book

Included in this homework are exercises 1.6 and 1.45.

Exercise 1

This exercise is a rephrasing of exercise 1.9 in the book.

In a game you are playing, you have been given four envelopes containing money. You know that each of these envelopes contain a different amount, but this is all you know: in particular, although you know that there is only one envelope with the highest amount, you will not be able to recognise it just by opening it. To be certain of it, you would have to open all the others.

The rules of the game are as follows. You open the first envelope and look at the amount. You may choose to keep it, or you can choose to open at the second; but in the latter case, the first envelope is thrown into the fire, and there is no way to go back. You are then given the same choice with the second envelope, and, should you choose to discard it, the third. Of course, getting to the fourth and last envelope, you should keep it.

Since you are extremely greedy, you have no intention to settle for the second prize: better nothing than the shame of not scoring the highest. Here are a few strategies you could follow.

1. You keep the first envelope.
2. You open the first envelope, throw it in the fire, and take the next envelope whose amount is higher (no point in keeping one that is lower), or the last envelope if you have to.
3. You open and destroy the first two envelopes, and take the next envelope if its amount is higher than both, otherwise you take the last envelope.

¹Monday 2:30-3:30, Thursday 10:30-11:30, or by appointment.

Assuming the envelopes are given to you in a uniform random ordering, what are the probabilities of getting the highest amount for each of these strategies?

Exercise 2

In poker, a two-pair is a hand of five cards with precisely two pairs of cards of the same value and a different isolated card (2 sevens, 2 kings and 1 six for instance, but not 4 aces and 1 three nor 2 eights and 3 sevens). They can be in any order.

1. Out of all the possible ordered hands of 5 cards, how many are two-pairs?
2. What is the probability that an ordered hand of 5 cards drawn uniformly at random be a two-pair? Do you find the same result as example 1.23 in the book? Should you?

Exercise 3

In a class of 20 students, a group of 5 is selected uniformly at random to work on a gardening project.

1. You don't mind participating, but only if you are with your buddy Mike. What is the probability that you will be disappointed?
2. Your pals Mike and Sarah have bad blood since that night at O'Malley's. Maybe working on something together will have them patch it up? Knowing that you and Mike have been selected, what is the probability that Sarah has been selected as well?