Homework 6

March 19th
Due March 26th

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name and assignment number should be clearly visible (like on this document for example). It is due on Gradescope.

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not. I will be available at my office hours[1] if you need help.

Exercises from the book

Included in this homework are exercises 2.6, 2.8, 2.9, 2.10 and 2.11.

Exercise 1

A company sells 1,000 mechanical pieces to a client. Each of these pieces may be defective or not. For each of the following numbers N, give the name and parameter(s) of the corresponding distribution, and (a formula for) the probability $\mathbb{P}(N = 5)$.

1. Suppose exactly two pieces are defective. The client has a quality control service, but it is incredibly lazy. They will check only one piece at random, and say that the whole batch is either perfect or dreadful, according to the condition of the piece. N is the number of defects that quality control will detect.

2. Suppose that each piece is defective with a probability 0.1%, independently of the others. N is the number of items with a defect.

3. As above, suppose that each piece is defective with a probability 0.1%, independently of the others. For the next contract (same model, same quantity, same probability of defect), the seller wants to be sure that no piece is defective. She will personally inspect each and every item, and throw away the defective ones. N is the number of pieces, defective or not, that the company will produce.

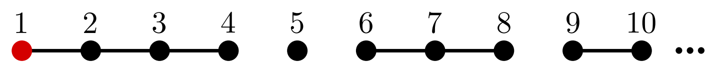
[1] Monday 2:30-3:30, Thursday 10:30-11:30, or by appointment.

Exercise 2

A minigolf champion always has the same probability to get the ball to the hole in one strike, wherever the two of them may be. In average, it takes him 1.5 strikes to clear a given obstacle. What is the probability that he finishes a 18 hole round in exactly 20 strikes?

Exercise 3

On a half-line, see below, we plot regularly spaced points; think of it as $\mathbb{N}^*$ if you like. Between two consecutive points, we draw an edge with probability $0 < p < 1$, independently from each other, and we can go from point to point only if there is an edge between these two.



1. Call N the index of the farthest point we can reach, starting from the leftmost point. During class, we gave a name to the distribution of N; what is it?
2. Show that the probability that we can go from 1 to infinity is zero.

   Hint: the event 'We can reach infinity' is included in the event 'We can reach 10,000'.
3. Consider the same model, but on the whole axis $\mathbb{Z}$ (infinite in both directions). What is the probability that we can go from zero to infinity, either to the left or to the right?