Homework 7

March 26th
Due April 5th

Your assignment may be handwritten or typeset, but in any case it should be neat and readable. Your name and assignment number should be clearly visible (like on this document for example). It is due on Gradescope.

You are encouraged to work in groups for this assignment. However, the redaction should be done on one's own: do not copy some other student's work, or give your assignment to some other student. To consult textbooks or online resources is fair game; on the other hand, to look up the exact exercise and its solution is not. I will be available at my office hours[1] if you need help.

Exercises from the book

Included in this homework are exercises 2.12, 2.14, 2.16.

Exercise 1

A random variable X takes the following values:

$$\mathbb{P}(X=-1)=\frac{1}{4}, \qquad \mathbb{P}(X=0)=\frac{1}{12}, \qquad \mathbb{P}(X=0.4)=\mathbb{P}(X=1)=\frac{1}{3}.$$

1. What is the expectation of X? Its variance?
2. Compute the expectation $\mathbb{E}[\cos(5\pi X)]$.

Exercise 2

Let N be a hypergeometric variable of parameter $(w,b,n)=(3,2,3)$.

What is the mean of N? What is its tenth percentile?

Exercise 3

Let N the result of a die throw, so that $N \sim \mathcal{U}nif(\{1,\dots,6\})$.

1. What is the expectation of N?

[1] Monday 2:30-3:30, Thursday 10:30-11:30, or by appointment.

2. Given that the expectation $\mathbb{E}[N^2]$ is $91/6$, what is the variance of N?

3. You throw a die a hundred times, and adding all results you get 319. Using the rough approximation $X \approx \mathbb{E}[X] \pm \sqrt{\mathrm{Var}(X)}$ when X is a random variable, do you think the die might be biased?

Exercise 4

Suppose there are two variables X and Y that can only take the following values.

- $X = -1$ and $Y = 1$;
- $X = -1$ and $Y = -1$;
- $X = 2$ and $Y = y$

for some unknown y. For instance, it is impossible to have $X = 2$ and $Y = 1$ unless $y = 1$. Suppose moreover that the three possibilities are equally likely.

1. Find y such that $\mathrm{Cov}(X, Y) = 0$.

2. For the chosen y, is it true that X and Y are independent?