Topics

1 Probability spaces, events, conditioning

Key concepts.

- Probability spaces, events.

  What is a probability space? What properties does the probability function satisfy? What is an event? What do the intersection, union and complement of events mean? What is the inclusion-exclusion principle and how does one use it?

- Conditioning.

  What is the definition of a conditional probability? When are two events independents? How does one use the multiplication formula? What is the law of total probability, and how does one use it? What is a partition?

- Bayes' law.

  How does one apply Bayes' law? In what situations is it useful? How does it work with the law of total probability?

Book sections.

- 1.1 Probability spaces
- 1.3 Conditional probability, independence and Bayes' formula

Sample of exercises.

- Book: 1 to 3, 5, 17 to 22, 24 to 27, 29, 39, 40, 42, 43, 45 to 49, 51 in Chapter 1.
- Homework: 1, 2, 3 in Homework 1; 1, 2 in Homework 2.
- Review: A to L in Review 1.
- Midterms and quizzes: 1, 2 in Quiz 1; 1, 2, 5, 7, 8 in Midterm 1.

2 Counting

Key concepts.

- Uniform probability.

  How does one compute the probability of an event in a uniform probability space? What about conditional probabilities?

- Factorial, falling factorial, binomial and multinomial coefficients.

  What are they? What do they count? How are they related? How does one recognise which of these can be used in a give situation?

Book sections.

- 1.2 Finite sample spaces and counting

Sample of exercises.

- Book: 6 to 16 in chapter 1.
- Homework: 1, 2, 3 in Homework 4; 3 in Homework 6.
- Review: M to Q in Review 1.
- Midterms and quizzes: 3 in Quiz; 3, 4, 6, 9 in Midterm 1.

3 Discrete variables, multivariate discrete variables

Key concepts.

- Discrete variables, probability mass function.

  What is a discrete random variable? What is the probability mass function? What does the graph of a probability mass function look like? How do we compute it?

- Cumulative distribution functions, quantiles.

  How does one identify the probability mass function from the cumulative distribution function, and vice versa? What does the graph of a cumulative distribution function look like? What are the median, the quantiles, the quartiles? How does one find them using the cumulative distribution function?

- Usual distributions: uniform, Bernoulli, geometric, binomial, hypergeometric, negative binomial, Poisson.

  What are the main properties of the usual distributions (probability mass function, expectation, variance)? What do they represent? How does one recognise which distribution is appropriate for a given situation?

- Multivariate discrete variables.

  What is a multivariate discrete variable? What is the probability mass function, the cumulative distribution of a multivariate discrete variable? What does it mean for two or more discrete variables to be independent? How does one compute the expectation of a function depending on multiple discrete variables?

Book sections.

- 2.1 Random variables — Some general facts
- 2.2 Discrete random variables

  except 2.2.2 (generating functions), variances and moments in 2.2.3, Markov and Chebyshev inequalities in 2.2.4

- 3.1 Discrete random vectors
- 3.3 Multidimensional discrete random vectors

Sample of exercises.

- Homework: 1, 2 in Homework 6; 1, 2, 4 in Homework 7; 1, 2, 3 in Homework 8; 2 in Homework 11.
- Review: A to I in Review 2; O to R in Review 3.
- Midterm and quizzes: 1, 2, 3, 4 in Quiz 2; 1, 2, 4, 6, 7, 8 in Midterm 2.

4 Variance, moments, inequalities

Key concepts.

- Expectation, variance, covariance.

  What are the expectation, the variance of a random variable? What do they represent? What are their properties? How does one compute them? How do they relate to independence?

- Markov's inequality, Chebyshev's inequality.

  What are the moments of a random variable? How does one compute them? How can one use them to get bounds on probabilities? What are the Markov and Chebyshev inequalities? How do they differ? When should one use one rather than the other? How do we get inequalities for probabilities from the expectation of various functions?

Book sections.

- 2.2 Discrete random variables

  only variances and moments in 2.2.3, Markov and Chebyshev inequalities in 2.2.4

- 2.3 Continuous random variables

  only variances in 2.3.1, Markov and Chebyshev inequalities in 2.3.1.

- 6.1 The law of large numbers
- 6.2 The central limit theorem

Sample of exercises.

- Homework: 3 in Homework 7; 4, 5 in Homework 8.
- Review: J to M in Review 2.
- Midterm and quizzes: 3, 5 in Midterm 2.

5 Continuous variables, multivariate continuous variables

Key concepts.

- Continuous variables, probability density function.

  What is a continuous random variable? What is the probability density function? What does the graph of a probability mass function look like? What does it represent? How does one compute the probability of a given event involving such a variable?

- Cumulative distribution functions, quantiles.

  How does one identify the probability mass function from the cumulative distribution function, and vice versa? What does the graph of a cumulative distribution function look like? How does one find the median and the quantiles using the cumulative distribution function?

- Usual distributions: uniform, exponential, normal, (gamma, beta).[1]

  What are the main properties of the usual distributions (probability density function, expectation, variance)? What do they represent? How does one recognise which distribution is appropriate for a given situation?

- Transformation of continuous random variables.

  How does one compute the expectation of a function depending on a given continuous random variable? How does one find its density, its cumulative distribution function?

- Multivariate continuous variables.

  What is a multivariate continuous variable? What is the probability density function, the cumulative distribution of a continuous discrete variable? What does it mean for two or more continuous variables to be independent? How does one compute the probability of an event involving two or more continuous random variables?

Book sections.

- 2.1 Random variables — Some general facts
- 2.3 Continuous random variables

  except variances in 2.3.1, Markov and Chebyshev inequalities in 2.3.1

- 4.1 Two-dimensional continuous random vectors

  only the very beginning, definition of the joint probability density function.

Sample of exercises.

- Homework: 1, 2 in Homework 9; 1, 2, 3 in Homework 10; 1 in Homework 11.
- Review: A to N, S to Y in Review 3.
- Midterm and quizzes: 1, 2, 3, 4 in Quiz 3.

[1] You should know these last two distributions exist and be able to reach out rapidly for your notes on their properties. You do not need to know their definition by heart.